Robust Estimation of Orientation for Texture Analysis

Rein van den Boomgaard, Joost van de Weijer

Abstract— **In this paper we describe a robust estimator for the orientation of linear textures. The proposed estimator is the robust version of a well-known orientation estimator based on the *structure tensor* that is related to the covariance matrix of the ensemble of gradient vectors that are observed in a local neighborhood.**

The structure tensor follows from a least square estimation of the local orientation. Classical least squares estimators suffer from the fact that at the borders of texture regions the statistics of two regions are mixed leading to biased measurements (the orientation is 'smoothed'). The proposed robust estimator is shown to be able to deal with these mixed model statistics and to find the correct orientation. Some preliminary results for generated textures are presented.

Keywords— **Orientation measurement, texture, robust estimation, structure tensor.**

I. Introduction

ORIENTED patterns are found in many imaging applications, e.g. in fingerprint analysis, and in geophysical analysis of soil layers(see Fig. 1) . The classical technique to estimate the orientation of the texture is to look at the ensemble of luminance gradient vectors in a local neighborhood. In an image patch showing a stripe pattern in only one orientation we can clearly distinguish the orientation as the line cluster in gradient space perpendicular to the stripes (see Fig. 2(a-b)). A straightforward eigenvector analysis of the covariance matrix will perfectly reveal the orientation of the texture. The covariance matrix of the gradient vectors in an image neighborhood is often used to estimate the local orientation [1], [2], [3], [4]

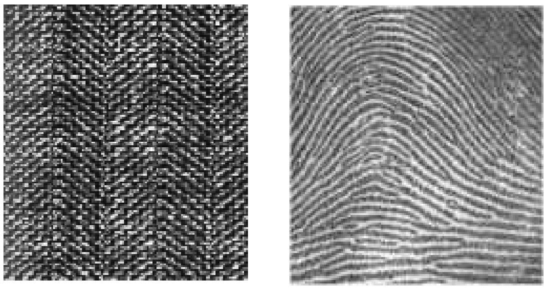

Fig. 1. **Oriented Patterns Examples.** Two examples of oriented patterns are shown. On the left the Herringbone fabric from the Brodatz set and on the right a fingerprint image.

In case the local neighborhood is taken from the border of two differently oriented patterns (see Fig. 2 an eigenvector analysis of the covariance matrix will mix both orientations resulting in a 'smoothing' of the orientation estimation.

Both authors are with the Intelligent Sensory Information Systems Group at the University of Amsterdam, The Netherlands. Email: {rein,joostw}@science.uva.nl.

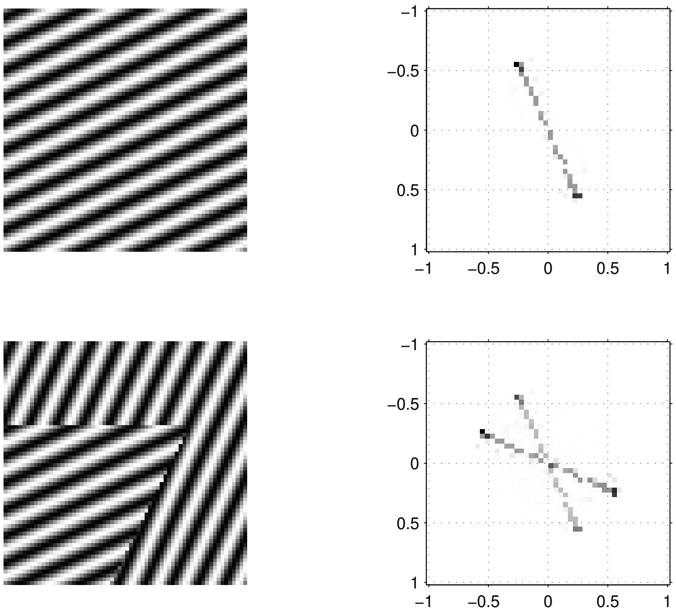

Fig. 2. **Histograms of gradient vector space.** In (a) an image (64 × 64) is shown with in (b) the histogram of all gradient vectors (where darker shades indicate that those gradient vectors occur often in the image. In (c) a composition of two differently oriented patterns is shown with corresponding histogram in (d).

In case the regions showing different textures are of sufficient size it is possible to use a post-processing step to sharpen the smoothed orientation measurements. A classical way of doing so is the Kuwahara-Nagao operator [5], [6], [7]. At a certain position in an image this operator searches for a nearby neighborhood where the (orientation) response is more homogeneous then it is at the border. That response is then used at the point of interest. In this way the neighborhoods are not allowed to cross the borders of the textured regions. In [8] we have shown that the classical Kuwahara-Nagao operator can be interpreted as a 'macroscopic' version of a PDE image evolution that combines linear diffusion (smoothing) with morphological sharpening.

A more principled solution would be to use a multi-texture model and estimate the orientation of all textures within the neighborhood. In such a model we would not only have to estimate the orientations but also the geometry that describes where the one pattern ends and the next one begins. The Heuckel edge detector [9] is the classical example of a model using two distributions (of luminance values) and estimating both the distribution parameters as well as the geometry separating the distributions spatially.

In this paper we take a less principled approach. Instead of a multi-texture model we will use *robust estimators* that allow us to use only the gradient vectors from one pattern in estimating the orientation and consider all other gradient vectors as statistical outliers.

The use of robust estimators is well known in image processing and computer vision. Their use has a long history (see [10], [11], [12]) but recently the connection to locally orderless images as introduced by Koenderink [13] and Grifin [14] described by Van de Weijer et al. [15]) and its use in non-linear diffusion (see Black [16]) has lead to a renewed interest in robust estimation techniques.

The use of robust estimators is shown in Section IV to result in less 'orientation smoothing' as is the case for the classical orientation estimation.

In Section II we show that the eigenvector analysis of the structure tensor is equivalent to a linear (total) least squares estimate of the local orientation. Having casted the orientation estimation in the form of least squares minimization problem, we are able to generalize it to a robust estimator (see Section III). The resulting iterative procedure in which each step determines the eigenvector of a 'robust covariance matrix' is shown to result in accurate and less smoothed orientation estimates.

II. Least Squares Estimation

Again consider the texture in Fig. 2(a). The histogram of the gradient vectors in this texture patch is shown in Fig. 2(b). Let $\mathbf{v}$ be the true orientation vector of the patch, i.e. the vector perpendicular to the stripes. In an ideal image patch every gradient vector should be parallel to the orientation $\mathbf{v}$. In practice they will not be parallel. The error of a gradient vector $\mathbf{g}(\mathbf{y})$ observed in a point $\mathbf{y}$ with respect to the orientation $\mathbf{v}(\mathbf{x})$ of an image patch centered at location $\mathbf{x}$ is defined as:

$$e(\mathbf{x},\mathbf{y}) = \|\mathbf{g}(\mathbf{y}) - (\mathbf{g}(\mathbf{y})^\top \mathbf{v}(\mathbf{x}))\mathbf{v}(\mathbf{x})\|$$

The difference $\mathbf{g}(\mathbf{y}) - (\mathbf{g}(\mathbf{y})^\top \mathbf{v}(\mathbf{x}))\mathbf{v}(\mathbf{x})$ is the projection of $\mathbf{g}$ on the normal to $\mathbf{v}$. The error $\epsilon(\mathbf{x},\mathbf{y})$ thus measures the perpendicular distance from the gradient vector to the orientation vector $\mathbf{v}$. Integrating the squared error over all positions $\mathbf{y}$ using a soft Gaussian aperture for the neighborhood definition we define the total error:

$$\epsilon(\mathbf{x}) = \int_\Omega e^2(\mathbf{x},\mathbf{y}) G^s(\mathbf{x}-\mathbf{y}) d\mathbf{y} \tag{1}$$

In order to get more manageable equations we omit the arguments for all functions in the above error measure:

$$\epsilon = \int_\Omega \|\mathbf{g} - (\mathbf{g}^\top \mathbf{v})\mathbf{v}\|^2 G^s d\mathbf{y}.$$

Using the fact the $\|\mathbf{a}\|^2 = \mathbf{a}^\top \mathbf{a}$ we rewrite this as:

$$\epsilon = \int_\Omega \left(\mathbf{g}^\top \mathbf{g} - (\mathbf{v}^\top (\mathbf{g}\mathbf{g}^\top)\mathbf{v})\right) G^s d\mathbf{y},$$

or equivalently

$$\epsilon = \int_\Omega \mathbf{g}^\top \mathbf{g} G^s d\mathbf{y} - \int_\Omega \mathbf{v}^\top (\mathbf{g}\mathbf{g}^\top)\mathbf{v} G^s d\mathbf{y}.$$

Minimizing the error thus is equivalent with maximizing:

$$\int_\Omega \mathbf{v}^\top (\mathbf{g}\mathbf{g}^\top)\mathbf{v} G^s d\mathbf{y},$$

subject to the constraint that $\mathbf{v}^\top \mathbf{v} = 1$. Note that $\mathbf{v}$ is not dependent on $\mathbf{y}$ so that we have to maximize:

$$\mathbf{v}^\top \left(\int_\Omega (\mathbf{g}\mathbf{g}^\top) G^s d\mathbf{y}\right) \mathbf{v} = \mathbf{v}^\top \mu^s \mathbf{v}$$

where μ^s is the *structure tensor*. The quadratic form achieves its maximal value in case we select for $\mathbf{v}$ the eigenvector of μ^s corresponding with the largest eigenvalue. We give a more formal proof here as we will need the derivation for our robust estimator in Section III.

Using the method of Lagrange multipliers to optimize subject to the constraint that $\mathbf{v}^\top \mathbf{v} = 1$, we need to find an extremum of

$$\lambda(1 - \mathbf{v}^\top \mathbf{v}) + \mathbf{v}^\top \mu^s \mathbf{v}.$$

Differentiating with respect to $\mathbf{v}$ (remember that $d\mathbf{v}^\top A\mathbf{v}/d\mathbf{v} = 2A\mathbf{v}$)and setting the derivative equal to zero results in:

$$\mu^s \mathbf{v} = \lambda \mathbf{v}. \tag{2}$$

The 'best' orientation thus is an eigenvector of the structure tensor. Substitution in the quadratic form then shows that we need the eigenvector corresponding to the largest eigenvalue.

III. Robust Estimation

The least squares orientation estimation works well in case all gradients in the ensemble of vectors in an image neighborhood all belong to the same oriented pattern. In case the image patch shows two oriented patterns the least squares estimate will mix the two orientations and give a wrong result.

The least squares estimator minimizes the error given in Eq. (1):

$$\epsilon(\mathbf{x}) = \int_\Omega \rho(e(\mathbf{x},\mathbf{y})) G^s(\mathbf{x}-\mathbf{y}) d\mathbf{y} \tag{3}$$

where we select the *error norm* $\rho(e) = e^2$. Large deviations from the model are thus very important in a least squares estimate. In a robust estimator large deviations from the model are not taken into account that heavily. The argument is that the deviation could be due to an outlier. In our application large deviations from the model are probably due to the mixing of two different linear textures (see Fig. 2(c-d)).

In this paper we use the Gaussian error norm:

$$\rho(e) = 1 - \exp(-\frac{e^2}{2t^2})$$

where t defines the 'width' of the error norm. In Fig. 3 the quadratic norm is compared with the Gaussian error norm. Observe that the influence of large errors is limited.

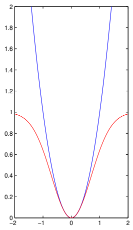

Fig. 3. **Quadratic versus (robust) Gaussian error norm.** The Gaussian error norm is of 'scale' $t = 0.7$.

The error, Eq. (3), can now be rewritten as (we will omit the spatial arguments):

$$\epsilon = \int_\Omega \rho \left(\sqrt{\mathbf{g}^\mathsf{T}\mathbf{g} - \mathbf{v}^\mathsf{T}(\mathbf{g}\mathbf{g}^\mathsf{T})\mathbf{v}} \right) G^s d\mathbf{y}.$$

Again we use a Lagrange multiplier to minimize subject to the constraint that $\mathbf{v}^\mathsf{T}\mathbf{v} = 1$:

$$\frac{d}{d\mathbf{v}} \left(\lambda(1 - \mathbf{v}^\mathsf{T}\mathbf{v}) + \int_\Omega \rho \left(\sqrt{\mathbf{g}^\mathsf{T}\mathbf{g} - \mathbf{v}^\mathsf{T}(\mathbf{g}\mathbf{g}^\mathsf{T})\mathbf{v}} \right) G^s d\mathbf{y} \right) = 0.$$

This leads to:

$$\eta(\mathbf{v})\mathbf{v} = \lambda \mathbf{v} \tag{4}$$

where

$$\eta(\mathbf{v}) = \frac{1}{2t^2} \int_\Omega \exp(-\frac{\mathbf{g}^\mathsf{T}\mathbf{g} - \mathbf{v}^\mathsf{T}(\mathbf{g}\mathbf{g}^\mathsf{T})\mathbf{v}}{2t^2}) \mathbf{g}\mathbf{g}^\mathsf{T} G^s d\mathbf{y}.$$

The big difference with the least squares estimator is that now the matrix η is dependent on $\mathbf{v}$ (and on $\mathbf{x}$ as well).

We propose the following *fixed point* iteration scheme to find a solution. Let $\mathbf{v}^i$ be the orientation vector estimate after i iterations. The estimate is then updated as the eigenvector $\mathbf{v}^{i+1}$ of the matrix $\eta(\mathbf{v}^i)$ corresponding to the largest eigenvalue, i.e. we solve:

$$\eta(\mathbf{v}^i)\mathbf{v}^{i+1} = \lambda \mathbf{v}^{i+1}$$

The proposed scheme is a generalization of the well-known fixed point scheme (also called *functional iteration*) to find a solution of the equation $v = F(v)$.

Note that the iterative scheme does not necessarily lead to the *global* minimum of the error. In fact often we are not even interested in that global minimum. Consider for instance the situation of a point in region A (with orientation α_1) that is surrounded by many points in region B (with orientation β). It is not to difficult to imagine a situation where the points of region B outnumber those in region A. Nevertheless we would like our algorithm to find the orientation α whereas the global minimum would correspond with orientation β. Because our algorithm starts in the initial orientation estimate and then finds the local minimum nearest to the starting point we hopefully end up in the desired *local* minimum: orientation α.

The choice for an initial estimate of the orientation vector is thus crucial in a robust estimator in case we have an image patch showing two (or more) striped patterns.

In the experiments shown in the next section the initial orientation estimate $\mathbf{v}^0$ is set to the orientation that is calculated with the least squares estimator.

IV. Experiments

In this section we present some preliminary experiments showing the improvement that robust orientation measurement brings compared to a least squares estimation.

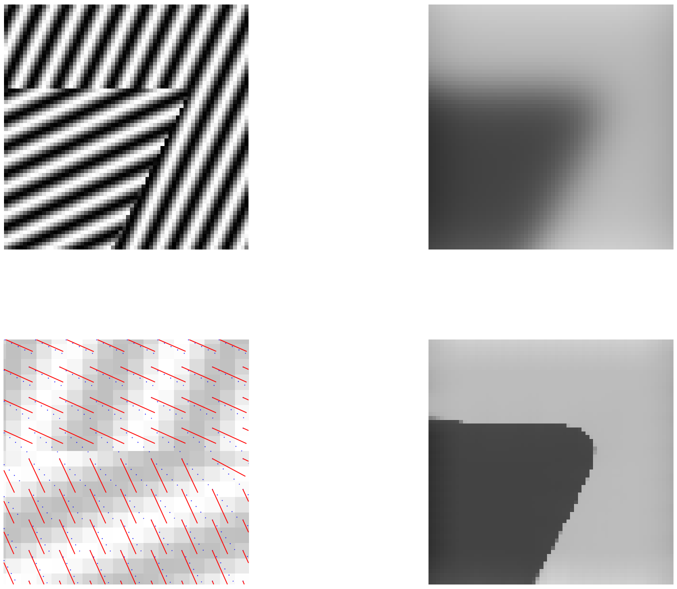

Fig. 4. **Least Squares versus Robust Orientation Estimation.** In (a) a generated noise free image is shown. The texture is made out of two regions each differently oriented. In (b) the orientation field $\alpha = \arctan(v_2/v_1)$ is shown that results from the least squares estimate. In (d) the orientation field is shown resulting from the robust estimation. In (c) a *detail* of the orientation vector fields for both the least squares estimation (dotted lines) and the robust estimation (solid lines) are shown.

In Fig. 5(a) a generated noise free image is shown. The texture is made out of two regions each differently oriented. In (b) the orientation field $\alpha = \arctan(v_2/v_1)$ is shown that results from the least squares estimate. In (d) the orientation field is shown resulting from the robust estimation. In (c) the orientation vector fields for both the least squares estimation (dotted lines) and the robust estimation (solid lines) are shown.

In Fig. 5(a) a generated image with some noise added is shown. The texture is made out of two regions each differently oriented. In (b) the orientation field $\alpha = \arctan(v_2/v_1)$ is shown that results from the least squares estimate. In (d) the orientation field is shown resulting from the robust estimation. In (c) the orientation vector fields for both the least squares estimation (dotted lines) and the robust estimation (solid lines) are shown.

For the robust estimation in both cases we have used the orientation in location $\mathbf{x}$ that resulted from the least squares estimator as the initial orientation vector in that point. In both cases only 5 iterations are used.

¿From both the noise free and the noise corrupted texture images it is evident that the robust estimation performs much better at the border of the textured regions.

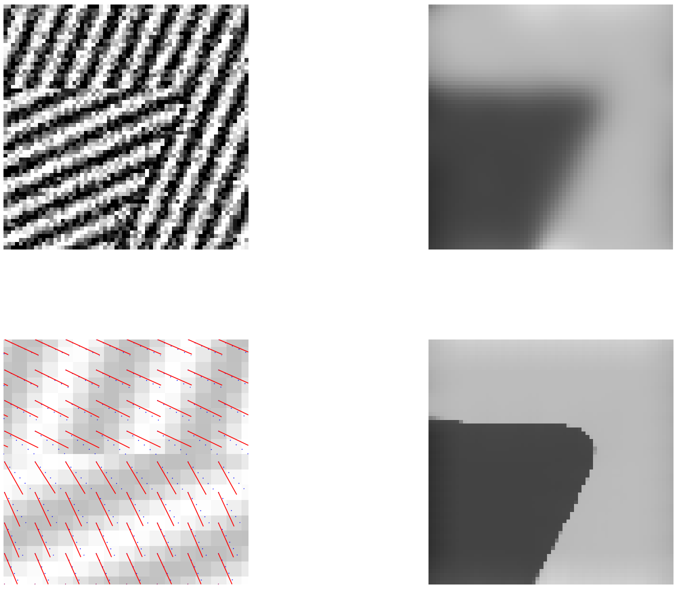

Fig. 5. **Least Squares versus Robust Orientation Estimation.** In (a) a generated image with some noise added is shown. The texture is made out of two regions each differently oriented. In (b) the orientation field $\alpha = \arctan(v_2/v_1)$ is shown that results from the least squares estimate. In (d) the orientation field is shown resulting from the robust estimation. In (c) a *detail* of the orientation vector fields for both the least squares estimation (dotted lines) and the robust estimation (solid lines) are shown.

V. Conclusions

In this paper we have shown that the use of robust estimation techniques result in texture measurements (orientation in our specific example) that suffer less from the mixing of two textures at the geometrical border of two differently textured regions.

The robust estimation method presented in this paper is based on fixed point (or functional) iteration. This view on robust estimation makes the link with mean shift analysis evident. This paper generalizes the robust estimators from the scalar case to the vectorial case. A more formal proof of the asymptotic result of the proposed iterative scheme is lacking in this paper and will be presented in another paper.

Here we have only considered linear texture models. It seems feasible to consider more elaborate stripe models as well. Weijer et al. have considered parabolic and circular shaped isophotes as well [17].

We believe that robust estimation techniques can not only be used to deal with statistical (noise) outliers but that in low-level vision an important usage will be to deal with multi-model image structure and texture without the need for explicit geometrical modeling of the image patches. In this way we can circumvent the need for post-processing to 'sharpen' the measurement responses.

References

[1] M. Kass and A. Witkin, "Analyzing oriented patterns," *Computer Graphics and Image Processing*, vol. 37, pp. 363–385, 1987.

[2] J. Bigun, G.H. Granlund, and J. Wiklund, "Multidimensional orientation estimation with application to texture analysis and optical flow," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 13, no. 8, pp. 775–789, 1991.

[3] T. Lindeberg and J. Garding, "Shape from texture from a multiscale perspective," Technical Report ISRN KTH/NA/P–93/03–SE, Royal Institution of Technology, Stockholm, 1993.

[4] J. Weickert, *Anisotropic Diffusion in Image Processing*, Tuebner Verlag, Stuttgart, 1997.

[5] M. Kuwahara, K. Hachimura, S. Eiho, and M. Kinoshita, "Processing of ri-angiocardiographic images," in *Digital Processing of Biomedical Images*, K. Preston and M. Onoe, Eds., 1976, pp. 187–202.

[6] M. Nagao and T. Matsuyama, "Edge preserving smoothing," *Computer Graphics and Image Processing*, vol. 9, no. 4, pp. 394–407, April 1979.

[7] P. Bakker, L.J. van Vliet, and P.W. Verbeek, "Edge preserving orientation adaptive filtering," in *ASCI'99, Proc. 5th Annual Conference of the Advanced School for Computing and Imaging*, M. Boasson, J.A. Kaandorp, J.F.M. Tonino, and M.G. Vosselman, Eds., 1999, pp. 207–213.

[8] R. van den Boomgaard, "The Kuwahara-Nagao operator decomposed in terms of a linear smoothing and a morphological sharpening," in *Mathematical Morphology, Proceedings of the 6th International Symposium on Mathematical Morphology*, H. Talbot and R. Beare, Eds., Sydney, Australia, April 2002, pp. 283–292, CSIRO Publishing.

[9] M.H. Heuckel, "An operator which locates edges in digital pictures," *J. Association for computing machinery*, vol. 18, pp. 113–125, 1971.

[10] P.J. Besl, J.B. Birch, and L.T. Watson, "Robust window operators," *Machine Vision and Applications*, vol. 2, pp. 179–191, 1989.

[11] P. Meer, D. Mintz, and A. Rosenfeld, "Robust regression methods for computer vision: A review," *Int. Journal of Computer Vision*, vol. 6, no. 1, pp. 59–70, 1991.

[12] C.V. Stewart, "Robust parameter estimation in computer vision," *SIAM Review*, vol. 41, no. 3, pp. 513–537, 1999.

[13] J.J. Koenderink and A.J. van Doorn, "The structure of locally orderless images," *Int. Journal of Computer Vision*, vol. 31, no. 2/3, pp. 159–168, 1999.

[14] L.D. Grifin, "Scale-imprecision space," *Image and Vision Computing*, vol. 15, pp. 369–398, 1997.

[15] J. van de Weijer and R. van den Boomgaard, "Robust ...," 2001.

[16] M. Black, G. Sapiro, D. Marimont, and D. Heeger, "Robust anisotropic diffusion," *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 421–432, 1998.

[17] J. van de Weijer, L.J. van Vliet, P. Verbeek, and M. van Ginkel, "Curvature estimation in oriented patterns using curvilinear models applied to gradient vector fields," *IEEE PAMI*, vol. 23, no. 9, pp. 1035–1042, 2001.